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Leptonic CP violation in flipped SU(5) GUT from $\mathbf{Z}_{12-I}$ orbifold compactification

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ABSTRACT

We obtain a phenomenologically acceptable PMNS matrix in a flipped SU(5) model, possessing the $\mathbf{Z}_{4R}$ discrete symmetry, from the compactification of heterotic string $\mathrm{E}_8 \times \mathrm{E}_8'$. To analyze the Jarlskog determinant efficiently, we include the simple Kim-Seo form for the Pontecorbo-Maki-Nakagawa-Sakata matrix. We also noted that $|\delta_{\rm PMNS}| \lesssim 63^{\rm o}$ for the normal hierarchy of neutrino masses with the PDG book parametrization.

1. Introduction

The most urgent theoretical issue in the standard model(SM) is probing the symmetry structure from which the observed flavor phenomena can be understood. It is desirable if such symmetry results from an ultra-violet completed theory. At present, string theory is considered to be the most attractive one among various ultra-violet completed theories, chiefly because it unifies gravity on the same ground as gauge theories. The SM is obtained from compactification of six extra dimensions [1–6], and the symmetry structure of flavors is the one realized below the compactification scale.

Previously, most studies along this direction were centered on obtaining three families in the standard-like models [7,8].[1] Since there are too many Yukawa couplings in standard-like models, here we attempt to work in a grand unification (GUT) models. The GUT group we use here is the rank 5 flipped SU(5) GUT [11,12], $\mathrm{SU(5)_{flip}} \equiv \mathrm{SU(5)} \times \mathrm{U(1)}_X$, which was obtained from compactifications via fermionic string [13] and $\mathbf{Z}_{12-I}$ orbifold [14]. The simplest GUT SU(5) from string compactification is not accompanying an adjoint representation at the level 1 construction where the Higgs multiplet needed for breaking SU(5) down to the SM is lacking. The rank 5 $\mathrm{SU(5)_{flip}}$ requires $\mathbf{10} \oplus \overline{\mathbf{10}}$ for breaking it down to the rank 4 SM gauge group, and the model of [10] contains them.

Time is ripe enough to study the details of the flavor structure from string compactification to see whether they converge to the observed data. The study is now possible in the $\mathbf{Z}_{4R}$ model [9] based on [10] where the needed $\mathbf{Z}_{4R}$ quantum numbers of *all light chiral fields* are presented. In the quark sector, the Cabibbo-Kobayashi-Maskawa matrix [15,16] has been studied in our previous paper [17]. In this paper, we present a numerical study on the Pontecorbo-Maki-Nakagawa-Sakata (PMNS) matrix [18,19] via many U(1)'s arising in string compactification. The heterotic string $\mathrm{E}_8 \times \mathrm{E}_8'$ has rank 16 gauge sector. The model presented in [10] has the $\mathrm{SU(5)_{flip}}$ from E_8 and

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[1] For more references, see [9].

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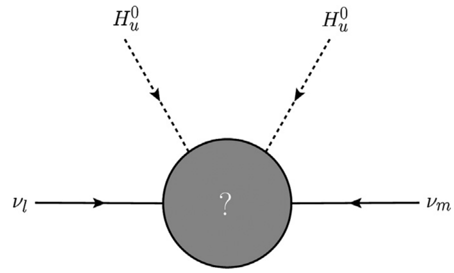

Fig. 1. A neutrino interaction with the SM fields only.

Table 1
U(1) charges of matter fields in the flipped SU(5). ξ_i and $\bar{\eta}_i$ contain the left-handed quark and lepton doublets, respectively, in the i-th family.

	State($P+kV_0$)	Θ_i	$\mathbf{R}_X$(Sect.)	Q_R	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$Q_{\rm anom}$	Q_{18}	Q_{20}	Q_{22}
ξ_3	$(\underline{+++--};--+)(0^8)'$	0	$\overline{\mathbf{10}}_{-1}(U_3)$	+1	−6	−6	+6	0	0	0	−13	+1	−1	+1
$\bar{\eta}_3$	$(\underline{+----};+--)(0^8)'$	0	$\mathbf{5}_{+3}(U_3)$	+1	+6	−6	−6	0	0	0	−1	+1	−1	+1
τ^c	$(+++++;-+-)(0^8)'$	0	$\mathbf{1}_{-5}(U_3)$	+1	−6	+6	−6	0	0	0	+5	+1	−1	+1
ξ_2	$(\underline{+++--};-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\overline{\mathbf{10}}_{-1}(T_4^0)$	−1	−2	−2	−2	0	0	0	−3	−1	−1	−1
$\bar{\eta}_2$	$(\underline{+----};-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\mathbf{5}_{+3}(T_4^0)$	−1	−2	−2	−2	0	0	0	−3	−1	−1	−1
μ^c	$(+++++;-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\mathbf{1}_{-5}(T_4^0)$	−1	−2	−2	−2	0	0	0	−3	−1	−1	−1
ξ_1	$(\underline{+++--};-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\overline{\mathbf{10}}_{-1}(T_4^0)$	−1	−2	−2	−2	0	0	0	−3	−1	−1	−1
$\bar{\eta}_1$	$(\underline{+----};-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\mathbf{5}_{+3}(T_4^0)$	−1	−2	−2	−2	0	0	0	−3	−1	−1	−1
e^c	$(+++++;-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\mathbf{1}_{-5}(T_4^0)$	−1	−2	−2	−2	0	0	0	−3	−1	−1	−1
H_{uL}	$(\underline{+1\,0\,0\,0\,0};\,0\,0\,0)(0^5;\,\frac{-1}{2}\,\frac{+1}{2}\,0)'$	$\frac{+1}{3}$	$2\cdot\mathbf{5}_{-2}(T_6)$	−2	0	0	0	−12	0	0	0	−1	−1	−1
H_{dL}	$(\underline{-1\,0\,0\,0\,0};\,0\,0\,0)(0^5;\,\frac{+1}{2}\,\frac{-1}{2}\,0)'$	$\frac{+1}{3}$	$2\cdot\overline{\mathbf{5}}_{+2}(T_6)$	+2	0	0	0	+12	0	0	0	−1	−1	−1

SU(5)$'\times$SU(2)$'$ from E$_8'$, and we can consider 6 extra U(1) gauge groups. These many U(1)'s make it possible to have flavor dependent Yukawa couplings.[2]

String compactification in our example allows all the needed Yukawa couplings in the SM as non-renormalizable forms. There has been an ambitious attempt [21] to relate the origin of the μ term with the magnitudes of neutrino masses by introducing just one singlet chiral field beyond the minimal supersymmetric SM. This try allowed only renormalizable couplings. Therefore, with so many singlets participating through non-renormalizable Yukawa couplings in our model, this design is not applicable. The relations in our model might be intertwined in an elaborate way since the $\mathbf{Z}_{4R}$ discrete symmetry automatically gives the μ term at the electroweak scale [22]. Since our $\mathbf{Z}_{4R}$ is a subgroup of an Abelian gauge group U(1)^6, it cannot be a non-Abelian discrete symmetry such as the interesting A_4 symmetry [23].

The mass matrices leading to the Kim-Seo (KS) parametrization [24] of the CKM and PMNS mixing matrices have a small number of complex elements, by making one row consist of real numbers. At the places where complex entries are allowed, we allocate the CP phase. The KS form has another advantage that the Jarlskog determinant is $J = -{\rm Im}\, V_{31}^{\rm KS} V_{22}^{\rm KS} V_{13}^{\rm KS}$ [25]. Toward a model building, the next step is to obtain phenomenologically acceptable mass matrices. In the flipped SU(5), the neutrino mass matrix is symmetric, can be made real, and hence we propose in this paper to put the CP phase in the charged lepton mass matrix.

The string model gives the Yukawa couplings for the charged lepton masses and neutrino masses. Thus, in Sec. 2 we present the lepton mass matrices from the flipped SU(5) model, *i.e.* allowed by the quantum numbers of Ref. [9], in the forms related to the KS mixing matrix. Then, we locate possible phases in the complex vacuum expectation values (VEVs) of the SM singlet fields σ_i. Next in Sec. 3, we relate these leptonic mass matrices with the PMNS matrix and compare it with the data presented in the Particle Data Book [26]. In Sec. 4, we present numerical analyses and obtain $|\delta_{\rm PMNS}| < 62.8^{\rm o}$ for the normal hierarchy of neutrino masses with the PDG book parametrization [26]. Section 5 is a conclusion. The magnitude on the weak CP violation J [27] is briefly discussed in Appendix A.

2. Suggestion from the flipped SU(5) model

If we consider only the SM particles, neutrino masses arise from the diagram shown in Fig. 1. Any further attachments to this diagram are SM singlet scalars. If we consider the quantum numbers under SU(2)$_W\times$U(1)$_Y$, two neutrinos have $\mathbf{1}_{-1}\oplus\mathbf{3}_{-1}^{\uparrow}$ where $\uparrow$ means that the 3rd component of the weak isospin is $+1$. Possible additional scalar attachments to Fig. 1 must carry quantum number $\mathbf{1}_{+1}$ or $\mathbf{3}_{+1}^{\downarrow}$ together with two H_u^0's, and $\mathbf{1}_{+1}$ is ruled out because $\langle\mathbf{1}_{+1}\rangle$ breaks U(1)$_{\rm em}$. $\mathbf{3}_{+1}^{\downarrow}$ allows the scalar attachments, shown as $H_u\oplus H_u$ in Fig. 1. Depending on details of high energy fields, implied by the question mark in the gray, two types of neutrino masses are named, Type I seesaw [28] and Type II seesaw [29]. Type III seesaw [30] requires more light particles at the electroweak scale. From the SU(5)$_{\rm flip}$ spectra shown in Ref. [20], we note that there is no SU(2) triplet representation; hence only Type I seesaw is allowed from our string compactification.

[2] Among these, the anomalous U(1) can work as a flavor symmetry [20]. Note that in addition to these we use U(1)'s from the extra dimensions toward $\mathbf{Z}_{4R}$ discrete symmetry in Ref. [9].

Table 2
U(1) charges of L-handed neutral scalars (but $\sigma_{7,8}$ for R-handed). We kept up to one oscillator represented as *Number of resulting fields*(number of oscillating mode). For example, $n(1_{\bar{1}})$ means that there results n multiplicities with one oscillator $\frac{-5}{12}$. For $Q_{18,20,22}$ charges, here we listed only those of L-handed fields, participating in the Yukawa couplings. $\sigma_{2,3,4,11',15',21,22,23,24}$ have phase $\Theta_i = 0$, which can be used to break $\mathbf{Z}_{4R}$ down to $\mathbf{Z}_{2R}$.

	State$(P+kV_0)$	Θ_i	$(N^L)_j$	$\mathcal{P}\cdot\mathbf{R}_X$(Sect.)	Q_R	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$Q_{\rm anom}$	Q_{18}	Q_{20}	Q_{22}
Σ_1^*	$(\underline{+++--};0^3)(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	0	$2(1_1)$	$2\,\overline{\mathbf{10}}_{-1}(T_3)_L$	+4	0	0	0	0	+9	+3	$\frac{-33}{7}$	−1	+1	−1
Σ_1^*	$(\underline{+++--};0^3)(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	$\frac{+2}{3}$	$1(1_3)$	$1\,\overline{\mathbf{10}}_{-1}(T_3)_L$	+4	0	0	0	0	+9	+3	$\frac{-33}{7}$	−1	+1	−1
Σ_2	$(\underline{++---};0^3)(0^5;\frac{+1}{4}\,\frac{+1}{4}\,\frac{-2}{4})'$	0	$2(1_{\bar{1}})$	$2\,\mathbf{10}_{+1}(T_3)_L$	−4	0	0	0	0	−9	−3	$\frac{+33}{7}$	−1	−1	−1
Σ_2	$(\underline{++---};0^3)(0^5;\frac{+1}{4}\,\frac{+1}{4}\,\frac{-2}{4})'$	$\frac{+1}{3}$	$1(1_{\bar{3}})$	$1\,\mathbf{10}_{+1}(T_3)_L$	−4	0	0	0	0	−9	−3	$\frac{+33}{7}$	−1	−1	−1
σ_1	$(0^5;\frac{-2}{3}\,\frac{-2}{3}\,\frac{-2}{3})(0^8)'$	$\frac{+1}{4}$	0	$2\cdot\mathbf{1}_0(T_4^0)$	−4	−8	−8	−8	0	0	0	−12	−1	−1	−1
σ_2	$(0^5;\frac{-2}{3}\,\frac{+1}{3}\,\frac{+1}{3})(0^8)'$	0	$3(1_{\bar{1}})$	$3\cdot\mathbf{1}_0(T_4^0)$	0	−8	+4	+4	0	0	0	−2	−1	−1	−1
σ_3	$(0^5;\frac{1}{3}\,\frac{-2}{3}\,\frac{1}{3})(0^8)'$	0	$3(1_{\bar{1}})$	$3\cdot\mathbf{1}_0(T_4^0)$	0	+4	−8	+4	0	0	0	−8	−1	−1	−1
σ_4	$(0^5;\frac{1}{3}\,\frac{1}{3}\,\frac{-2}{3})(0^8)'$	0	$3(1_{\bar{1}})$	$3\cdot\mathbf{1}_0(T_4^0)$	0	+4	+4	−8	0	0	0	+10	−1	−1	−1
σ_5	$(0^5;0\,1\,0)(0^5;\frac{1}{2}\,\frac{-1}{2}\,0)'$	$\frac{+1}{2}$	0	$2\cdot\mathbf{1}_0(T_6)$	+4	0	+12	0	+12	0	0	+14	−1	−1	−1
σ_6	$(0^5;0\,0\,1)(0^5;\frac{-1}{2}\,\frac{1}{2}\,0)'$	$\frac{+1}{2}$	0	$2\cdot\mathbf{1}_0(T_6)$	0	0	0	+12	−12	0	0	−4	−1	−1	−1
σ_7	$(0^5;0\,-1\,0)(0^5;\frac{-1}{2}\,\frac{1}{2}\,0)'$	$\frac{+1}{2}$	0	$2\cdot\mathbf{1}_0(T_6)_R$	+4	0	+12	0	+12	0	0	+14	−1	+1	−1
σ_8	$(0^5;0\,0\,-1)(0^5;\frac{1}{2}\,\frac{-1}{2}\,0)'$	$\frac{+1}{2}$	0	$2\cdot\mathbf{1}_0(T_6)_R$	−2	0	0	+12	−12	0	0	−4	−1	+1	−1
σ_{11}	$(0^5;\frac{-1}{2}\,\frac{-1}{2}\,\frac{-1}{2})(0^5;\frac{3}{4}\,\frac{-1}{4}\,\frac{-1}{2})'$	$\frac{+2}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	−6	−6	−6	−6	+12	−9	−3	$\frac{-30}{7}$	+1	+1	−1
σ'_{11}	$(0^5;\frac{-1}{2}\,\frac{-1}{2}\,\frac{-1}{2})(0^5;\frac{3}{4}\,\frac{-1}{4}\,\frac{-1}{2})'$	0	$4(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$4\cdot\mathbf{1}_0(T_3)$	−6	−6	−6	−6	+12	−9	−3	$\frac{-30}{7}$	−1	+1	+1
σ_{12}	$(0^5;\frac{-1}{2}\,\frac{1}{2}\,\frac{1}{2})(0^5;\frac{3}{4}\,\frac{-1}{4}\,\frac{-1}{2})'$	$\frac{+1}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	−2	−6	+6	+6	+12	−9	−3	$\frac{+40}{7}$	+1	+1	−1
σ'_{12}	$(0^5;\frac{-1}{2}\,\frac{1}{2}\,\frac{1}{2})(0^5;\frac{3}{4}\,\frac{-1}{4}\,\frac{-1}{2})'$	$\frac{+2}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	−2	−6	+6	+6	+12	−9	−3	$\frac{+40}{7}$	−1	+1	+1
σ_{13}	$(0^5;\frac{1}{2}\,\frac{1}{2}\,\frac{-1}{2})(0^5;\frac{-1}{4}\,\frac{3}{4}\,\frac{-1}{2})'$	$\frac{+1}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	−6	+6	+6	−6	−12	−9	−3	$\frac{+124}{7}$	+1	+1	−1
σ'_{13}	$(0^5;\frac{1}{2}\,\frac{1}{2}\,\frac{-1}{2})(0^5;\frac{-1}{4}\,\frac{3}{4}\,\frac{-1}{2})'$	$\frac{+2}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	−6	+6	+6	−6	−12	−9	−3	$\frac{+124}{7}$	−1	+1	+1
σ_{14}	$(0^5;\frac{1}{2}\,\frac{1}{2}\,\frac{-1}{2})(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{1}{2})'$	$\frac{+2}{3}$	$2(1_{\bar{1}})+1(1_{\bar{3}})$	$3\cdot\mathbf{1}_0(T_3)$	+4	+6	+6	−6	0	+9	+3	$\frac{+58}{7}$	−1	+1	+1
σ_{15}	$(0^5;\frac{-1}{2}\,\frac{-1}{2}\,\frac{-1}{2})(0^5;\frac{+3}{4}\,\frac{-1}{4}\,\frac{-1}{2})'$	$\frac{+2}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	−6	−6	−6	−6	+12	−9	−3	$\frac{-30}{7}$	+1	+1	−1
σ'_{15}	$(0^5;\frac{-1}{2}\,\frac{-1}{2}\,\frac{-1}{2})(0^5;\frac{+3}{4}\,\frac{-1}{4}\,\frac{-1}{2})'$	0	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$4\cdot\mathbf{1}_0(T_3)$	−6	−6	−6	−6	+12	−9	−3	$\frac{-30}{7}$	−1	+1	+1
σ_{16}	$(0^5;\frac{-1}{2}\,\frac{+1}{2}\,\frac{+1}{2})(0^5;\frac{+3}{4}\,\frac{-1}{4}\,\frac{-1}{2})'$	$\frac{+1}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	−2	−6	+6	+6	+12	−9	−3	$\frac{+40}{7}$	+1	+1	−1
σ'_{16}	$(0^5;\frac{-1}{2}\,\frac{+1}{2}\,\frac{+1}{2})(0^5;\frac{+3}{4}\,\frac{-1}{4}\,\frac{-1}{2})'$	$\frac{+2}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	−2	−6	+6	+6	+12	−9	−3	$\frac{+40}{7}$	−1	+1	+1
σ_{17}	$(0^5;\frac{+1}{2}\,\frac{+1}{2}\,\frac{-1}{2})(0^5;\frac{-1}{4}\,\frac{+3}{4}\,\frac{-1}{2})'$	$\frac{+1}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	−6	+6	+6	−6	−12	−9	−3	$\frac{+124}{7}$	+1	+1	−1
σ'_{17}	$(0^5;\frac{+1}{2}\,\frac{+1}{2}\,\frac{-1}{2})(0^5;\frac{-1}{4}\,\frac{+3}{4}\,\frac{-1}{2})'$	$\frac{+2}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	−6	+6	+6	−6	−12	−9	−3	$\frac{+124}{7}$	−1	+1	+1
σ_{18}	$(0^5;\frac{1}{2}\,\frac{+1}{2}\,\frac{-1}{2})(0^5;\frac{+3}{4}\,\frac{-1}{4}\,\frac{-1}{2})'$	$\frac{+2}{3}$	$2(1_{\bar{1}})+1(1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	+4	+6	+6	−6	0	+9	+3	$\frac{+58}{7}$	−1	+1	+1
σ_{21}	$(0^5;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^5;\frac{1}{4}\,\frac{1}{4}\,\frac{-1}{2})'$	0	$1(1_{\bar{1}})$	$\mathbf{1}_0(T_1^0)$	+2	−2	−2	−2	0	+9	+3	$\frac{+12}{7}$	−1	−1	−1
σ_{22}	$(0^5;\frac{-5}{6}\,\frac{1}{6}\,\frac{1}{6})(0^5;\frac{1}{4}\,\frac{1}{4}\,\frac{-1}{2})'$	0	$1(1_{\bar{1}}+1_3)$	$\mathbf{1}_0(T_5^0)$	+2	−10	+2	+2	0	+9	+3	$\frac{-2}{7}$	−1	+1	+1
σ_{23}	$(0^5;\frac{1}{6}\,\frac{-5}{6}\,\frac{1}{6})(0^5;\frac{1}{4}\,\frac{1}{4}\,\frac{-1}{2})'$	0	$1(1_{\bar{1}}+1_3)$	$\mathbf{1}_0(T_5^0)$	+2	−10	+2	+2	0	+9	+3	$\frac{-44}{7}$	−1	+1	+1
σ_{24}	$(0^5;\frac{1}{6}\,\frac{1}{6}\,\frac{-5}{6})(0^5;\frac{1}{4}\,\frac{1}{4}\,\frac{-1}{2})'$	0	$1(1_{\bar{1}}+1_3)$	$\mathbf{1}_0(T_5^0)$	+2	−10	+2	+2	0	+9	+3	$\frac{+82}{7}$	−1	+1	+1

The SM fields from [9] are shown in Table 1, and the SM singlet fields, including those in $\overline{\mathbf{10}}_{-1}$ and $\mathbf{10}_{+1}$ of SU(5)$_{\rm flip}$, are shown in Table 2. Considering the SM singlet attachments to Fig. 1, let us consider the neutrino mass operators allowed by the quantum numbers of Tables 1 and 2. Firstly, the diagonal masses are

$$\begin{aligned}
M^\nu_{33} &\propto \frac{1}{\tilde{M}_3^3}\int d^2\vartheta\; \mathbf{5}_{+3}(U_3,0;+1)\mathbf{5}_{+3}(U_3,0;+1)\mathbf{5}_{-2}(T_6,\frac{1}{3};-2)\mathbf{5}_{-2}(T_6,\frac{1}{3};-2)\overline{\mathbf{10}}_{-1}(T_3,\frac{1}{3};+4)\overline{\mathbf{10}}_{-1}(T_3,0;+4)\\
M^\nu_{22} &\propto \frac{1}{\tilde{M}_2^4}\int d^2\vartheta\; \mathbf{5}_{+3}(T_4^0,\frac{1}{4};-1)\mathbf{5}_{+3}(T_4^0,\frac{1}{4};-1)\mathbf{5}_{-2}(T_6,\frac{1}{3};-2)\mathbf{5}_{-2}(T_6,\frac{1}{3};-2)\overline{\mathbf{10}}_{-1}(T_3,\frac{1}{3};+4)\overline{\mathbf{10}}_{-1}(T_3,0;+4)\\
&\qquad\cdot\mathbf{1}_0(\sigma_5,T_6,\frac{1}{2};+4)
\end{aligned}\tag{1}$$

where the last number after ; is the Q_R charge, and $\tilde{M}_3$ and $\tilde{M}_2$ are determined by ? in Fig. 1. We need $Q_R = 2$ modulo 4 above for $d^2\vartheta$ integration. $M^\nu_{11}, M^\nu_{12}, M^\nu_{21}$ have the same structure as M^ν_{22}. Note that the selection rule is making the phase an integer multiple, which is satisfied above for Θ_i, viz. $0+0+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+0=1$ and $\frac{1}{4}+\frac{1}{4}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+0+\frac{1}{2}=2$. Then, the above masses are estimated as

$$M^\nu_{33}\sim\frac{v_{EW}^2 M_{10}^2}{\tilde{M}_3^3},\; M^\nu_{22}\sim\frac{v_{EW}^2 M_{10}^2|\langle\sigma_5\rangle|}{\tilde{M}_2^4},\tag{2}$$

where $M_{10}=\langle\overline{\mathbf{10}}_{-1}\rangle=\langle\mathbf{10}_{+1}\rangle$. Then, neutrino mixing masses are generally of order $v_{EW}^2/\tilde{M}$ since the SM singlet VEV $\langle\sigma_5\rangle$ can be at the GUT scale without breaking $\mathbf{Z}_{4R}$.

For the off-diagonal masses between U_3 and T_4^0 neutrinos, we need $Q_R = 0$ modulo 4 for $d^2\vartheta d^2\bar{\vartheta}$ integration.

$$\begin{aligned}
M^\nu_{32}, M^\nu_{31} \propto &\frac{1}{\tilde{M}^6}\int d^2\vartheta d^2\bar{\vartheta}\; \mathbf{5}_{+3}(U_3,0;+1)\mathbf{5}_{+3}(T_4^0,\frac{1}{4};-1)\mathbf{5}_{-2}(T_6,\frac{1}{3};-2)\mathbf{5}_{-2}(T_6,\frac{1}{3};-2)\\
&\cdot\overline{\mathbf{10}}_{-1}(T_3,0;+4)\overline{\mathbf{10}}_{-1}(T_3,0;+4)\cdot\mathbf{1}_0(\sigma_1,T_4^0,\frac{1}{4};-4)^*\mathbf{1}_0(\sigma_{14},T_3,\frac{2}{3};+4)^*.
\end{aligned}\tag{3}$$

Then, the above mass mixing is estimated as

$$M^{\nu}_{13,23} \sim \frac{v^2_{EW} M^2_{10} |\langle \sigma_1 \sigma_{14} \rangle|}{\tilde{M}^5_{\times}}, \tag{4}$$

where $\tilde{M}_{\times}$ is some mass scale determined by the above equations. Note that Σ^*_2, Σ_1 and σ_1 can have the GUT scale VEVs because all of them carry $Q_R = 0$ modulo 4, and we obtain a similar order of mass for all of $M^{\nu}_{11,12,22,33,13,23}$.

Comparing $M^{\nu}_{11,22,31,32}$ and M^{ν}_{33},

$$\frac{M^{\nu}_{11}, M^{\nu}_{22}}{M^{\nu}_{33}} \approx \left| \frac{\sigma_5}{\tilde{M}} \right|, \; \frac{M^{\nu}_{31}, M^{\nu}_{32}}{M^{\nu}_{33}} \approx \left| \frac{\langle \sigma_1 \sigma_{14} \rangle}{\tilde{M}^2} \right|, \tag{5}$$

we note that the neutrino mass hierarchy can be the normal hierarchy (in the sense that ν_τ is the heaviest) if the VEVs of σ singlets are comparably small, $|\sigma_1|, |\sigma_5| < \tilde{M}$.

Since we obtained all entries in the neutrino mass matrix, here we investigate how the CP phase can be inserted in the mass matrix of the $Q_{\rm em} = -1$ leptons and in the neutrino mass matrix.

2.1. Neutrino mass matrix inspired by flipped SU(5)

In Ref. [9] based on the flipped SU(5) model of [10], a possible identification $\mathbf{Z}_{4R}$ has been achieved, forbidding dimension-5 B violating operators but allowing the electroweak scale μ term and dimension-5 L violating Weinberg operator. In the flipped SU(5), the neutrino masses arise in the form

$$-\mathcal{L}^{IJ}_{\nu} = f^{(\nu)}_{IJ}(\{\sigma\}) \mathbf{5}^{I,i}_{+3} \mathbf{5}^{J,j}_{+3} \mathbf{5}^{k}_{-2}(H_u) \mathbf{5}^{l}_{-2}(H_u) [\overline{\mathbf{10}}_{-1}(H_{\rm GUT}) \overline{\mathbf{10}}_{-1}(H_{\rm GUT})]_{ijkl} + \text{h.c.}, \tag{6}$$

where the couplings $f^{(\nu)}_{IJ}$ are complex parameters, I and J are flavor indices, i, j, k, l, m are SU(5) indices, and the subscript is the U(1)$_X$ quantum number of SU(5)$_{\rm flip}$. $\mathbf{5}_{-2}$ is usually denoted as H_{uL}, and $\overline{\mathbf{10}}_{-1}$, together with $\mathbf{10}_{+1}$, is the ten-plet needed for breaking the rank 5 gauge group SU(5)×U(1) at a GUT scale down to the rank 4 SM gauge group.

Consider $\mathbf{5}^{I,i}_{+3} \mathbf{5}^{J,j}_{+3}$ in Eq. (6) which is symmetric under I and J. Thus, the neutrino mass matrix is symmetric. The Majorana phase factored out in Eq. (20) is from the heavy neutrinos, which will not affect our study of CC interactions of Sec. 3. We assume that the neutrino mass matrix, being symmetric, is real. Thus, $V^{(\nu)}$ can be considered to be an orthogonal matrix $O^{(\nu)}$.

2.2. Mass matrix of charged leptons inspired by flipped SU(5)

As commented above, we can always take $V^{(\nu)}$ as a real matrix $O^{(\nu)}$. Thus, the PMNS matrix given in the KS form, Eq. (25), can be represented as

$$V^{(l)\dagger}_{\rm KS} = V^{(e)} O^{(\nu)T} = \begin{pmatrix} q_{11}r_{11} + q_{12}r_{12} + q_{13}r_{13}, & q_{11}r_{21} + q_{12}r_{22} + q_{13}r_{23}, & q_{11}r_{31} + q_{12}r_{32} + q_{13}r_{33}, \\ q_{21}r_{11} + q_{22}r_{12} + q_{23}r_{13}, & q_{21}r_{21} + q_{22}r_{22} + q_{23}r_{23}, & q_{21}r_{31} + q_{22}r_{32} + q_{23}r_{33}, \\ q_{31}r_{11} + q_{32}r_{12} + q_{33}r_{13}, & q_{31}r_{21} + q_{32}r_{22} + q_{33}r_{23}, & q_{31}r_{31} + q_{32}r_{32} + q_{33}r_{33}, \end{pmatrix} \tag{7}$$

where the elements $V^{(l)}_{ij} = q_{ij}$ and $O^{(\nu)}_{ij} = r_{ij}$ are complex and real numbers, respectively. Making the 1st row real for the KS form, q_{11}, q_{12} and q_{13} are required to be real.

The unitary matrices relating the weak eigenstates l amd mass eigenstates i of the charged leptons are named as V for L-handed fields and U for R-handed fields,

$$l_L = \sum_{j=1}^{3} V^{(l)}_{lj} l^{(\rm mass)}_{jL}, \; l_R = \sum_{j=1}^{3} U^{(l)}_{lj} l^{(\rm mass)}_{jR}, \tag{8}$$

where $\bar{l}^{\rm mass}_L = (\ell_1, \ell_2, \ell_3)_L$ in terms of mass eigenstates ℓ_1, ℓ_2, ℓ_3, and $l^{\rm mass}_R = (N_1, N_2, N_3)_R$. The mass matrix becomes

$$\bar{l}_L (V^{(l)}_e M^{\rm mass} U^{(l)\dagger}_e) l_R \tag{9}$$

in the weak eigenstate basis. Since R-handed leptons are not participating in the CC interactions, the lepton R-handed unitary matrix $U^{(l)}$ can be taken as the identity matrix. Thus, the mass matrix in the weak basis becomes

$$M^{(l)} = V^{(l)} \begin{pmatrix} \tilde{m}_e, & 0, & 0 \\ 0, & \tilde{m}_\mu, & 0 \\ 0, & 0, & 1 \end{pmatrix} U^{(l)\dagger} = \begin{pmatrix} q_{11}\tilde{m}_e, & q_{12}\tilde{m}_\mu, & q_{13} \\ q_{21}\tilde{m}_e, & q_{22}\tilde{m}_\mu, & q_{23} \\ q_{31}\tilde{m}_e, & q_{32}\tilde{m}_\mu, & q_{33} \end{pmatrix} = \begin{pmatrix} \text{real}, & \text{real}, & \text{real} \\ \text{complex}, & \text{complex}, & \text{complex} \\ \text{complex}, & \text{complex}, & \text{complex} \end{pmatrix} \tag{10}$$

where $V^{(l)}_{ij} = q_{ij}$ and we obtained q_{11}, q_{12} and q_{13} are real numbers.

We show that the quantum numbers of the model presented in [9] allows an effective mass matrix form Eq. (10) for the charged leptons.

$$-\mathcal{L}^{IJ}_{l} = f^{(l)}_{IJ}(\{\sigma\}) \mathbf{5}^{I,i}_{+3} \mathbf{1}^{J}_{-5} \overline{\mathbf{5}}_{+2,i}(H_d) [\mathbf{10}_{+1}(H_{\rm GUT}) \overline{\mathbf{10}}_{-1}(H_{\rm GUT})] + \text{h.c.}, \tag{11}$$

which arises from, for example for the (22), (33) and (32) elements, viz. Tables 1 and 2,

$$\begin{aligned}
&\frac{1}{M^4}\int d^2\vartheta d^2\bar{\vartheta}\ \bar{\eta}_2(T_4^0,\frac{1}{4};-1)\mu^c(T_4^0,\frac{1}{4};-1)H_d(T_6,\frac{1}{3};+2)\Sigma_2(T_3,0;-4)\Sigma_1^*(T_3,\frac{2}{3};+4)\sigma_5^*(T_6,-\frac{1}{2};-4)\\
&\frac{1}{M^4}\int d^2\vartheta d^2\bar{\vartheta}\ \bar{\eta}_3(U_3,0;+1)\tau^c(U_3,0;+1)H_d(T_6,\frac{1}{3};+2)\Sigma_2(T_3,0;-4)\Sigma_1^*(T_3,\frac{2}{3};+4)\sigma_2^*(T_4^0,0;0)\\
&\frac{1}{M^4}\int d^2\vartheta\ \bar{\eta}_3(U_3,0;+1)\mu^c(T_4^0,\frac{1}{4};-1)H_d(T_6,\frac{1}{3};+2)\Sigma_2(T_3,0;-4)\Sigma_1^*(T_3,\frac{2}{3};+4)\sigma_1(T_4^0,\frac{1}{4};-4)\sigma_5(T_6,\frac{1}{2};+4)
\end{aligned}\tag{12}$$

which are allowed because they satisfy $Q_R=0$ (needed for D-terms) and $Q_R=2$ (needed for F-terms) modulo 4, respectively. The BSM fields in Eq. (12) carry $Q_R=4$ modulo 4 and hence $\mathbf{Z}_{4R}$ is not broken by the mass terms of the charged leptons.

Since all the entries of the mass matrix $M^{(l)}$ are allowed, we show below how the required form (10) results. Because of the degeneracy of the SM fields in the sector T_4^0, the mass matrix can be written as

$$\sim\begin{pmatrix} r_1e^{i\phi_1}, & r_1e^{i\phi_1}, & r_2e^{i\phi_2}\\ r_1e^{i\phi_1}, & r_1e^{i\phi_1}, & r_2e^{i\phi_2}\\ r_3e^{i\phi_3}, & r_3e^{i\phi_3}, & r_4e^{i\phi_4}\end{pmatrix}.\tag{13}$$

Redefining the L-handed and R-handed phases,

$$l'_L=\begin{pmatrix} e^{i\phi_5}, & 0, & 0\\ 0, & e^{i\phi_5}, & 0\\ 0, & 0, & 1\end{pmatrix} l_L,\ l'_R=\begin{pmatrix} 1, & 0, & 0\\ 0, & 1, & 0\\ 0, & 0, & e^{i\phi_6}\end{pmatrix} l_R,\tag{14}$$

we obtain the mass matrix for the choice of $\phi_5=-\phi_1$ and $\phi_6=\phi_2-\phi_1$,

$$\sim\begin{pmatrix} r_1, & r_1, & r_2\\ r_1, & r_1, & r_2\\ r_3e^{i\phi_3}, & r_3e^{i\phi_3}, & r_4e^{-i\phi_2}\end{pmatrix}.\tag{15}$$

The above mass matrix form is simple enough to assign phases in the SM singlet fields, σ_i. From Eq. (12), we can choose the following phase for the singlets, $\langle\sigma_1\rangle\sim e^{i\phi_3}$, $\langle\sigma_5\rangle\sim e^{i\cdot 0}$ and $\langle\sigma_2\rangle\sim e^{i\phi_2}$. Determining these phases is postponed until a sufficiently accurate value of the PMNS phase is known.

3. Diagonalization of mass matrices and mixing angles in the KS form

The charged current (CC) coupling in the lepton sector is

$$\mathcal{L}_{\rm CC}=-\frac{g}{\sqrt{2}}\sum_{l=e,\mu,\tau}\bar{l}_L\gamma^\alpha\nu_{lL}W_\alpha^-+\text{h.c.}\tag{16}$$

where the weak eigenstate leptons l are the defining ones in the CC interaction, and the weak eigenstate leptons l,ν_l are related to the mass eigenstate leptons $l_i^{(\rm mass)}$, $\nu_i^{(\rm mass)}$ as

$$l_L=\sum_{j=1}^{3}V_{lj}^{(e)}l_{jL}^{(\rm mass)},\ \ \nu_{lL}=\sum_{j=1}^{3}V_{lj}^{(\nu)}\nu_{jL}^{(\rm mass)},\tag{17}$$

where the first equation is given already in Eq. (8). The neutrino masses presented in Sec. 2 are in the weak eigenstate bases. Between the mass eigenstates, the CC interaction is given by

$$\mathcal{L}_{\rm CC}=-\frac{g}{\sqrt{2}}\bar{l}_L^{(\rm mass)}\gamma^\alpha V^{(e)\dagger}V^{(\nu)}\nu_L^{(\rm mass)}W_\alpha^-+\text{h.c.}\tag{18}$$

The PMNS matrix is given by[3]

$$V_{\rm PMNS}^\dagger=V^{(e)\dagger}V^{(\nu)}\tag{19}$$

where $V^{(e)}$ and $V^{(\nu)}$ are diagonalizing unitary matrices of L-handed charged leptons and neutrino fields.

A standard way to parametrize the CC lepton interactions is

$$\text{CC lepton matrix}=V_{\rm PMNS}^\dagger\times\begin{pmatrix}1, & 0 & 0\\ 0 & e^{i\alpha_{21}/2} & 0\\ 0 & 0 & e^{i\alpha_{31}/2}\end{pmatrix}\tag{20}$$

where the first factor called the PMNS matrix is usually written as [31,26]

[3] Compare with the CKM matrix $V_{\rm CKM}=V^{(u)\dagger}V^{(d)}$ defined from the W_μ^+ coupling, $-\frac{g}{\sqrt{2}}\bar{u}_L^{(\rm mass)}\gamma^\alpha V^{(u)\dagger}V^{(d)}d_L^{(\rm mass)}W_\alpha^++\text{h.c.}$

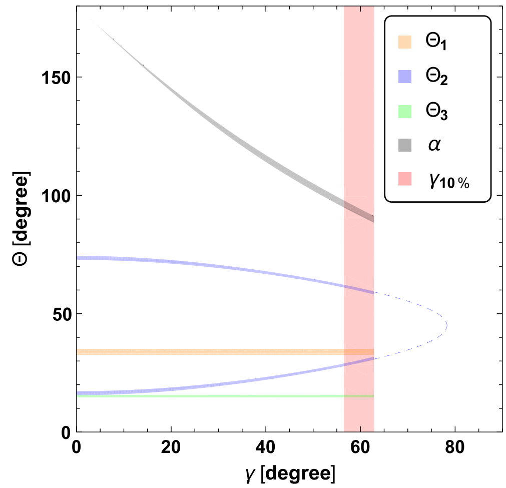

Fig. 2. The KS values of $\Theta_{1,2,3}$ and α in the first quadrant within one sigma bounds of the PDG values as functions in the allowed region of γ. In the second quadrant, $\Theta_{1,2,3}$ are given as $\Theta_{1,2,3}(-\gamma)=\Theta_{1,2,3}(\gamma)$. In the third quadrant α is given as $\alpha(-\gamma)\simeq-\alpha(\gamma)$. The 10% allowed region from the maximum, $\gamma=[+62.8^{\rm o},+56.56^{\rm o}]$, is shown as the pink band. The solution for Θ_2 is an ellipse whose tangent, as shown by the extended dashed curve, gives the limit determining the maximum $|\gamma|$'s in Eq. (23).

$$V^\dagger_{\rm PMNS}\simeq\begin{pmatrix} C_{12}C_{13}, & S_{12}C_{13}, & S_{13}e^{-i\delta}\\ -S_{12}C_{23}-C_{12}S_{23}S_{13}e^{i\delta}, & C_{12}C_{23}-S_{12}S_{23}S_{13}e^{i\delta}, & S_{23}C_{13}\\ S_{12}S_{23}-C_{12}C_{23}S_{13}e^{i\delta}, & -C_{12}S_{23}-S_{12}C_{23}S_{13}e^{i\delta}, & C_{23}C_{13}\end{pmatrix}, \tag{21}$$

where $C_{ij}=\cos\Theta_{ij}$, $S_{ij}=\sin\Theta_{ij}$, $\Theta_{ij}=[0,\frac{\pi}{2})$, and the angle $\delta=[0,2\pi]$ is the Dirac CP violation phase, and α_{21},α_{31} are two Majorana CP violation phases. The second factor of (20) contains the Majorana phases which may be determined by heavy neutrinos in the seesaw mechanism. The best fit (BF) real angles of the PMNS matrix are [26],

$$\begin{aligned}&\Theta_{12}=0.5764\ [C_{12}=0.8385,\ S_{12}=0.5450],\\ &\Theta_{23}=0.7101\ [C_{23}=0.7583,\ S_{23}=0.6519],\\ &\Theta_{13}=0.1472\ [C_{13}=0.9892,\ S_{13}=0.1466].\end{aligned}\tag{22}$$

and we have the following bound from Fig. 2,

$$-78.29^{\rm o}{}^{+2.35^{\rm o}}_{-0.90^{\rm o}}<\gamma<+78.29^{\rm o}{}^{+0.90^{\rm o}}_{-2.35^{\rm o}} \tag{23}$$

from which we obtain

$$\begin{aligned}&V^\dagger_{\rm PMNS}\simeq\begin{pmatrix} 0.8294, & 0.5391, & 0.1466\,e^{-i\delta}\\ -0.4132-0.08015\,e^{i\delta}, & 0.6358-0.0521\,e^{i\delta}, & 0.6449\\ 0.3553-0.0932\,e^{i\delta}, & -0.5466-0.0606\,e^{i\delta}, & 0.7501\end{pmatrix},\\ &\qquad J={\rm Im}\,V_{11}V_{22}V_{33}=-3.28\times10^{-2}\sin\delta\end{aligned}\tag{24}$$

where we used the central values for the allowed angles, $\theta_{12}=0.5758(=32.99^{\rm o})$, $\theta_{13}=0.1471(=8.428^{\rm o})$ and $\theta_{23}=0.7101(=40.69^{\rm o})$, for the normal hierarchy[4] of neutrino masses $m_1<m_2<m_3$.

The mass matrix in SU(5)$_{\rm flip}$ from string was presented in Sec. 2 in the KS form, where the 1st row of the PMNS matrix is made real [24],

$$V^\dagger_{\rm KS}=\begin{pmatrix} C_1, & S_1C_3, & S_1S_3\\ -C_2S_1, & C_1C_2C_3+S_2S_3e^{-i\delta_{\rm KS}}, & C_1C_2S_3-S_2C_3e^{-i\delta_{\rm KS}}\\ -e^{i\delta_{\rm KS}}S_1S_2, & -C_2S_3+C_1S_2C_3e^{i\delta_{\rm KS}}, & C_2C_3+C_1S_2S_3e^{i\delta_{\rm KS}}\end{pmatrix}, \tag{25}$$

where ${\rm Det}\,V_{\rm KS}=1$. Then, the phase appearing in the (31) element is the key, viz. $J=-{\rm Im}\,V^{\rm KS}_{31}V^{\rm KS}_{22}V^{\rm KS}_{13}=-C_1C_2C_3S_1^2S_2S_3\sin\delta_{\rm KS}$ [25]. So, let us obtain data in the KS form from the data in the PDG book [26]. In view of Fig. 5, we solve the equations for θ_i in terms of θ_{ij},

$$\begin{aligned}&c_1s_1s_3=c_{12}c_{13}s_{13},\\ &c_2^2c_3^2s_1^2s_2^2+c_1^2s_1^2s_3^2-2c_1c_2c_3s_1^2s_2s_3\cos\alpha=c_{13}^2c_{23}^2s_{12}^2s_{23}^2,\\ &c_2^2c_3^2s_1^2s_2^2=c_{13}^2c_{23}^2s_{12}^2s_{23}^2+c_{12}^2c_{13}^2s_{13}^2-2c_{12}c_{13}^2c_{23}s_{12}s_{23}s_{13}\cos\gamma.\end{aligned}\tag{26}$$

[4] Here we cite, for simplicity of presentation, mainly the numbers for normal hierarchy of neutrino masses except in Fig. 3.

For the fixed triangle given by (26), the area relation results in

$$c_2c_3s_1s_2 \sin\alpha = c_{13}c_{23}s_{12}s_{23} \sin\gamma. \tag{27}$$

Since there are four parameters to be determined *i.e.* $\theta_{1,2,3}$ and α from Eq. (26), there is a degree of freedom to define the KS form from the observed angles in the PDG book. Even if we can determine the KS parameters from (26) with one degeneracy parameter, the additional relation (27) has a profound meaning. It must be satisfied for all real values of parameters θ_i, α and θ_{ij}, γ. For some angles, therefore, there must be a bound for the relation (27) to be satisfied. Let us fix the parametrization such that the (11) element in the KS form agrees with the (11) element of the PDG book, $c_1 = c_{12}c_{13}$. Then, the four conditions to determine the KS parameters are

$$\begin{aligned}
&c_1 = c_{12}c_{13},\\
&s_1s_3 = s_{13},\\
&\sqrt{c_2^2c_3^2s_1^2s_2^2 + c_1^2s_1^2s_3^2 - 2c_1c_2c_3s_1^2s_2s_3\cos\alpha} = \pm c_{13}c_{23}s_{12}s_{23},\\
&c_2c_3s_1s_2 = \pm\sqrt{c_{13}^2c_{23}^2s_{12}^2s_{23}^2 + c_{12}^2c_{13}^2s_{13}^2 - 2c_{12}c_{13}^2c_{23}s_{12}s_{23}s_{13}\cos\gamma}.
\end{aligned} \tag{28}$$

The second relation of (28) is the important parameter in the neutrino oscillation and hence the condition for the (11) element to reproduce the PDG's (11) element is intuitive and persuasive. From the known values of θ_{ij} and the solutions of (26), γ should be bounded. Especially, it cannot be $-\frac{\pi}{2}$.

4. Numerical analyses

In this section, we present numerical analyses for the KS form angles of the PMNS matrix. The best fit (BF) real angles from [26] determine Θ_1 and Θ_3 accurately,

$$\begin{aligned}
&\Theta_1 = 0.5928\ [C_1 = 0.8294,\ S_1 = 0.5587],\\
&\Theta_3 = 0.2656\ [C_3 = 0.9649,\ S_3 = 0.2625],
\end{aligned} \tag{29}$$

but Θ_2 can be 0.5377 or 1.0331. Numbers given in Eq. (29) are presented in Fig. 2.[5] For $\alpha = -\frac{\pi}{2}$ (corresponding to $\gamma = -62.8^{\rm o}$) and $\Theta_2 = 0.5377$, we have

$$\begin{aligned}
&V_{\rm KS}^\dagger = \begin{pmatrix} 0.82939, & 0.53909, & 0.14663\\ -0.47985, & 0.68740 + 0.13441e^{-i\delta_{\rm KS}}, & 0.18697 - 0.49417e^{-i\delta_{\rm KS}}\\ -0.28611e^{i\delta_{\rm KS}}, & -0.22543 + 0.40986e^{i\delta_{\rm KS}}, & 0.82880 + 0.11148e^{-i\delta_{\rm KS}} \end{pmatrix},\\
&\quad J = -{\rm Im}\, V_{31}^{\rm KS}V_{22}^{\rm KS}V_{13}^{\rm KS} = -2.8838\times 10^{-2}\sin\delta_{\rm KS}.
\end{aligned} \tag{30}$$

Namely, to have J given in Eq. (30) for $\delta_{\rm KS} = -\frac{\pi}{2}$ compared to J of Eq. (24), we have the *minimum allowed value* $\gamma = -62.8^{\rm o}$ which is inside the region given in Eq. (23). In Fig. 2, we mark the $+10\%$ band from this value, $\gamma = [62.8^{\rm o}, 56.52^{\rm o}]$, as the pink band. In the third quadrant, the band becomes anti-symmetric to the curve in the first quadrant, $\gamma = [-62.8^{\rm o}\ {}^{+1.25^{\rm o}}_{-1.56^{\rm o}}, -56.56]$. In Fig. 3, we present an inverted hierarchy solution for $m_3 < m_1 < m_2$.

We used † notation in Eqs. (25) and (30) since the definition of the PMNS matrix is given by W_μ^- coupling and the CKM matrix is given by W_μ^+ coupling. To compare both with the W_μ^+ coupling, factoring out the Majorana phases, let us consider the PMNS parametrization with † of Eq. (25),

$$V_{\rm KS} = \begin{pmatrix} C_1, & -C_2S_1, & -S_1S_2e^{-i\delta_{\rm KS}}\\ S_1C_3, & C_1C_2C_3 + S_2S_3e^{i\delta_{\rm KS}}, & -C_2S_3 + C_1S_2C_3e^{-i\delta_{\rm KS}}\\ S_1S_3, & C_1C_2S_3 - S_2C_3e^{i\delta_{\rm KS}}, & C_2C_3 + C_1S_2S_3e^{-i\delta_{\rm KS}} \end{pmatrix}, \tag{31}$$

To build a model, leading to (31), one must find out the mass matrices $M^{(\nu)}$ and $M^{(l)}$ with appropriate insertions of $e^{\pm i\delta_{\rm KS}}$, which has been already shown in Sec. 2.

As suggested in [32], if we use the KS parametrization for the CKM matrix given in [24] and again the KS parametrization for the PMNS matrix given in Eq. (31) and the same CP phase α appears in the CKM and PMNS phases, we expect the unitary triangles take the forms given in Fig. 4 (a). If we use the Maiani-Chau-Keung (MCK) parametrization for the CKM matrix and the Schechter-Valle (SV) parametrization for the PMNS matrix given in [31,26] and the same CP phase γ appears in the CKM and PMNS phases, we expect the unitary triangles take the forms given in Fig. 4 (b). The CKM unitary triangle is known rather accurately but the PMNS unitary triangle is not known accurately, chiefly because the error bars allowed for γ is large: e.g. for the normal hierarchy $\delta_{\rm CP} = -1.728^{+0.851}_{-0.855}$ [34]. These unitary triangles are defined by CC interactions, and determined chiefly by the decay processes in the quark sector and by neutrino oscillations in the lepton sector.

[5] An approximate analytic solution near the dodeca symmetric point was given before [33].

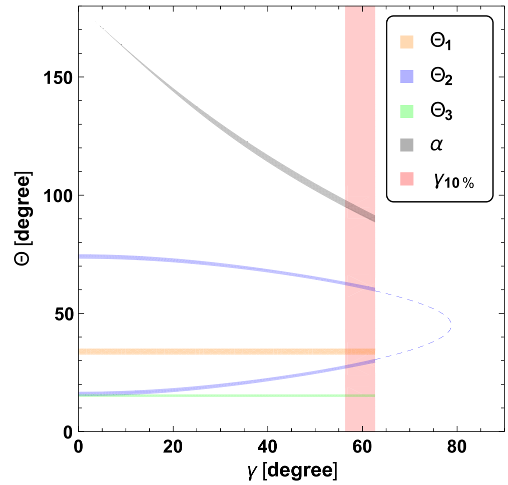

Fig. 3. Same as in Fig. 2 except for the inverted hierarchy [26], $m_3 < m_1 < m_2$, where Eq. (23) becomes $-78.80^{o}{}^{+0.94^o}_{-3.01^o} < \gamma < +78.80^{o}{}^{+3.01^o}_{-0.94^o}$ and the lower limit of γ becomes $-62.64^{o}{}^{+1.46^o}_{-1.48^o} < \gamma$.

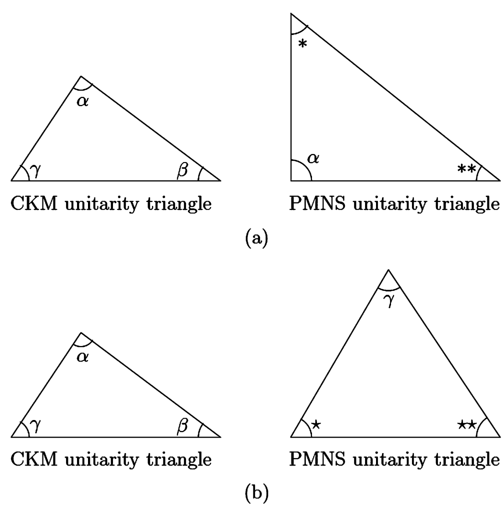

Fig. 4. The CKM and PMNS unitary triangles with one common angle [32]: (a) α in the KS form, and (b) γ in the SV form.

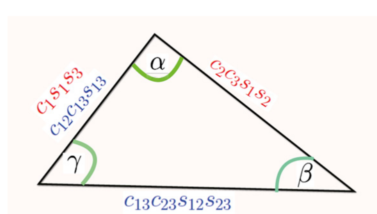

Fig. 5. The Jarlskog triangle [27,35] where $c_i = \cos\theta_i, s_i = \sin\theta_i$, and $c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}$.

5. Conclusion

We obtained the mass matrix forms of neutrinos and charged leptons from symmetries allowed in a compactified string [9], possessing $\mathbf{Z}_{4R}$ discrete symmetry. Discrete R parity $\mathbf{Z}_{4R}$ is crucial in supersymmetric SMs toward the solutions of the dimension-5 proton decay problem, μ problem, and acceptable neutrino masses [22]. In the flipped SU(5), the neutrino mass matrix is symmetric, and the weak CP phase is inserted in the mass matrix of charged leptons. This is then related to the Kim-Seo form of the PMNS matrix. The flipped SU(5) model compactified on $\mathbf{Z}_{12-I}$ is simple enough to draw this analysis up to satisfying all data on the PMNS matrix.

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Appendix A. Jarlskog determinant

In this Appendix, we comment on the Jarlskog determinant. For three families in the SM, the W boson couplings couple to the charged currents (CCs) in terms of four angles (three real angles and one phase) in the quark sector [16] and also in the lepton sector [18,19]. The weak CP violation in the SM is given by the Jarlskog determinant J. The numerical value of J is twice the area of the triangle shown in Fig. 5. Originally, J was given by a sum of four products of V_{ij} in the form V^*VV^*V [27]. It was proved that by making Det. V real, one can express it in terms of three products of V_{ij}: $J = -\mathrm{Im}\, V^{\rm KS}_{31}V^{\rm KS}_{22}V^{\rm KS}_{13}$ or $J = \mathrm{Im}\, V^{\rm KS}_{11}V^{\rm KS}_{22}V^{\rm KS}_{33}$, etc. [25]. In this form, J relates the entire range of the 3×3 matrix, covering all three families, and hence it can be a theory dependent number. So, it is better to use this form of J. Fig. 5 is drawn by considering $\sum_i V^*_{i1}V_{i3} = 0$.

One intuitive parametrization is using three rotation angles Θ_{ij} in the (ij) planes, with cosines $V_{ii} = V_{jj} = c_{ij}$ and sines $V_{ij} = s_{ij}, V_{ji} = -s_{ij}$ in V_{ij} and $V_{kk} = 1$ for $k|\epsilon_{ijk}|$ in the plane (ij). A non-Majorana phase δ is inserted conveniently in (13) plane [36]: $V_{1j} = (c_{13}, 0, s_{13}e^{-i\delta})$, $V_{2j} = (0, 1, 0)$, $V_{3j} = (-s_{13}e^{i\delta}, 0, c_{13})$. Since the unitary matrix in the (13) plane has the real determinant, here also we can use $J = -\mathrm{Im}\, V^{\rm KS}_{31}V^{\rm KS}_{22}V^{\rm KS}_{13}$ or $J = \mathrm{Im}\, V^{\rm KS}_{11}V^{\rm KS}_{22}V^{\rm KS}_{33}$, etc.

The angles α, β, and γ, in the unitarity relation $\sum_i V^*_{i1}V_{i3} = 0$, of Fig. 5 are related to δ of V. The same area of the triangle can be given in terms of α, β, or γ. For the parameters of Fig. 5, J can be expressed as $(c_1s_1s_3)(c_2c_3s_1s_2)\sin\alpha$ for the parameters of [24] or $(c_{12}c_{13}s_{13})(c_{13}c_{23}s_{12}s_{23})\sin\gamma$ for the parameters of [36]. The usefulness of the Kim-Seo (KS) parametrization with Det. $V^{\rm KS} = 1$ is making one row real, e.g. in the first row [24],

$$V_{\rm KS} = \begin{pmatrix} R_1, & R_2, & R_3 \\ T_1, & R_4 + R_5 e^{-i\delta}, & T_2 \\ R_6 e^{i\delta}, & T_3, & T_4 \end{pmatrix}, \tag{A.1}$$

where R_i and T_i are real and complex numbers, respectively. Then, we have $J = -R_3R_4R_6\sin\delta$. In this form, it is possible to visualize the CP phase $e^{i\delta}$ appearing in the position V_{31} [25]. A complex (22) element can be always written in the form separating out the term with the factor $e^{-i\delta}$.

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